EXERCISE – III

SUBJECTIVE QUESTIONS

1. Find the equation to the hyperbola whose directrix is $2x + y = 1$, focus $(1, 1)$ & eccentricity $\sqrt{3}$. Find also the length of its latus rectum.

Sol.

2. The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through the point of intersection of the lines $7x + 13y - 87 = 0$ and $5x - 8y + 7 = 0$ & the latus rectum is $32\sqrt{2}/5$. Find 'a' & 'b'.

Sol.

3. For the hyperbola $\frac{x^2}{100} - \frac{y^2}{25} = 1$, prove that

(i) eccentricity = $\sqrt{5}/2$

(ii) SA. S′A = 25, where S & S′ are the foci & A is the vertex.

Sol.

4. Find the centre, the foci, the directrices, the length of the latus rectum, the length & the equations of the axes of the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$.

Sol.

5. Find the equation of the tangent to the hyperbola $x^2 - 4y^2 = 36$ which is perpendicular to the line $x - y + 4 = 0$.

Sol.

6. Tangents are drawn to the hyperbola $3x^2 - 2y^2 = 25$ from the point (0, 5/2). Find their equations.

Sol.

7. If C is the centre of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, S, S′ its foci and P a point on it.

Prove that $SP.S'P = CP^2 - a^2 + b^2$.

Sol.

8. If θ_1 & θ_2 are the parameters of the extremities of a chord through (ae, 0) of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then show that $\tan\frac{\theta_1}{2}.\tan\frac{\theta_2}{2} + \frac{e-1}{e+1} = 0$.

Sol.

9. Tangents are drawn from the point (α, β) to the hyperbola $3x^2 - 2y^2 = 6$ and are inclined at angles θ and ϕ to the x-axis. If $\tan\theta.\tan\phi = 2$, prove that $\beta^2 = 2\alpha^2 - 7$.

Sol.

10. If two points P & Q on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ whose centre is C be such that CP is perpendicular to CQ & a < b, then prove that $\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} - \frac{1}{b^2}$.

Sol.

11. An ellipse has eccentricity 1/2 and one focus at the point P(1/2, 1). Its one directrix is the common tangent, nearer to the point P, to the circle $x^2 + y^2 = 1$ and the hyperbola $x^2 - y^2 = 1$. Find the equation of the ellipse in the standard form.

Sol.

12. The tangents & normal at a point on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cut the y-axis at A & B. Prove that the circle on AB as diameter passes through the foci of the hyperbola.

Sol.

13. The perpendicular from the centre upon the normal on any point of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets at R. Find the locus of R.

Sol.

14. If the normal at a point P to the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ meets the x-axis at G, show that SG = e.SP, S being the focus of the hyperbola.

Sol.

15. Show that the locus of the middle points of normal chords of the rectangular hyperbola $x^2 - y^2 = a^2$ is $(y^2 - x^2)^3 = 4\,a^2x^2y^2$.

Sol.

16. If a chord joining the points P $(a \sec\theta, a \tan\theta)$ & Q $(a \sec\phi, a \tan\phi)$ on the hyperbola $x^2 - y^2 = a^2$ is a normal to it at P, then show that

$\tan \phi = \tan \theta\,(4 \sec^2\theta - 1)$.

Sol.

17. Chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are tangents to the circle drawn on the line joining the foci as diameter. Find the locus of the point of intersection of tangents at the extremities of the chords.

Sol.

18. Let 'p' be the perpendicular distance from the centre C of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to the tangent drawn at a point R on the hyperbola. If S & S′ are the two foci of the hyperbola, then show that $(RS + RS')^2 = 4a^2\left(1+\frac{b^2}{p^2}\right)$.

Sol.

19. Prove that the part of the tangent at any point of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ intercepted between the point of contact and the transverse axis is a harmonic mean between the lengths of the perpendiculars drawn from the foci on the normal at the same point.

Sol.

20. An ellipse and a hyperbola have their principal axes along the coordinate axes and have a common foci separated by a distance $2\sqrt{13}$, the difference of their focal semi axes is equal to 4. If the ratio of their eccentricities is 3/7. Find the equation of these curves.

Sol.